MATH 1920, Fall 2017
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Sections: 201, 202

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Section 18.1 Review

Section 18.1 Summary

- State *Green's Theorem*:

- State the formulas for the area of the region D enclosed by C (using Green's Theorem):

- State the *Vector Form of Green's Theorem*:

Section 18.1 Additional Exercises

1. For Exercises (a) and (b), use Green's Theorem to evaluate the line integral. Orient the curve counterclockwise unless otherwise indicated.

   (a) $\oint_C \boldsymbol{F} \cdot dr$, where $\boldsymbol{F}(x, y) = \langle x + y, x^2 - y \rangle$ and C is the boundary of the region enclosed by $y = x^2$ and $y = \sqrt{x}$ for $0 \leq x \leq 1$.

   (b) $\oint_C \boldsymbol{F} \cdot dr$, where $\boldsymbol{F}(x, y) = \langle x^2, x^2 \rangle$ and C consists of the arcs $y = x^2$ and $y = x$ for $0 \leq x \leq 1$.

2. Let C_R be the circle of radius R centered at the origin. Use the general form of Green's Theorem to determine $\oint_{C_2} \boldsymbol{F} \cdot dr$, where F is a vector field such that $\oint_{C_1} \boldsymbol{F} \cdot dr = 9$ (C_1 is oriented counterclockwise, note how this changes when C_1 is considered as part of the boundary of the annulus) and $\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = x^2 + y^2$ for (x, y) in the annulus $1 \leq x^2 + y^2 \leq 4$.

3. Referring to Figure 25, suppose that

$$\oint_{C_2} \boldsymbol{F} \cdot dr = 3\pi, \oint_{C_3} \boldsymbol{F} \cdot dr = 4\pi$$

Use Green's Theorem to determine the circulation of $\boldsymbol{F}$ around C_1 assuming that $\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 9$ on the shaded region.

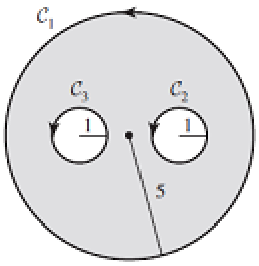

FIGURE 25

4. Let C_R be the circle of radius R centered at the origin. Use Green's Theorem to find the value of R that maximizes

$$\oint_{C_R} y^3 \; dx + x \; dy.$$

5. Use the vector form of Green's Theorem to find the flux of $\boldsymbol{F} = \langle 2x + y^3, 3y - x^4 \rangle$ across the boundary of the unit circle.

6. Do the same for $\boldsymbol{F} = \langle xy^2 + 2x, x^2y - 2y \rangle$ across the simple closed curve that is the boundary of the half disk given by $x^2 + y^2 \leq 3, y \geq 0$.

7. Let $\boldsymbol{F}$ be the velocity field. Estimate the circulation of $\boldsymbol{F}$ around a circle of radius $R = 0.05$ with center P, assuming that $\text{curl}_z(\boldsymbol{F})(P) = -3$. In which direction would a small paddle placed at P rotate?